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The Evidence for A Guessing State in Working-Memory Judgments

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Abstract

We present the results of four experiments designed to assess whether there is a guessing component to visual working memory, where guessing refers to responses that do not reflect the presented stimuli. The presence of guessing is important because such components are intimately consistent with discrete-slot view of WM and inconsistent with many competing views. We use the production paradigm from Zhang and Luck (2008, *Nature*, 453, 233-235), except our stimuli, which differ in angular displacement, are free to vary only within a restricted range rather than across all angles. As a result, guesses take on a distinctive pattern: when people guess they tend to select either a stereotypical clockwise or counter-clockwise position, and indeed this pattern is observed across all experiments. We formally fit a large number of discrete-slot and distributed-resource models and find that modified discrete-slot models outperform distributed-resource models in all experiments. One novel phenomenon we find is *coarse encoding*. Either items are encoded with fine-grain metric information, or they are encoded coarsely up to a categorical value (e.g., a stereotypical clockwise or counter-clockwise orientation). Implications for future theoretical developments are discussed.

KEYWORDS: Working Memory — Capacity — Working Memory Capacity

The Evidence for A Guessing State in Working-Memory Judgments

Working memory (WM) refers to the information available consciously at a given point in time. This information is limited in size and duration, though the nature of these limits is a point of theoretical tension. One well-known theoretical account is the *discrete-slots model*. Accordingly, working memory is composed of a fixed number of slots in which items or groups of items are temporarily held (Cowan, 1995; Miller, 1956). If there are more to-be-remembered items than slots, then some of these items will be represented in WM while others will not. Responses to items in memory will certainly reflect properties of the item, for example its orientation, color, or size. Responses to items not in memory, however, should not reflect such stimulus properties. Because these responses are invariant to item properties, they are indeed *guesses*. This invariance not only defines guesses, but is a signature of discrete-slot models.

An alternatives to the discrete-slot models are *distributed resource models*. According to these models, a limited amount of latent resources are divided and allocated to each item. The more resources allocated to an item, the better the performance to that item. The greater the number of to-be-remembered items, the less resources allocated to any one, and the worse the overall performance. Bays & Husain (2008) proposed a resource-based account similar in spirit to Navon & Gopher (1979) where the memory for an item is a function of the share of resources it receives. In their formal modeling, Bays and Husain proposed that each item receives an equal share of a fixed resource pool. More recently Sims, Jacobs, & Knill (2012) and van den Berg, Shin, Chou, George, & Ma (2012) generalized this approach where the resources are distributed unevenly and variably across items. In the strongest versions of the resource models, all items receive some share of the resources, and, consequently, performance to any item is never a guess (Bays & Husain, 2009). It is possible to formulate a continuous resource model that also includes guessing

(Sims et al., 2012, van den Berg et al., 2014) but such a model would seem unparsimonious inasmuch as a resource allocation to an item that can decrease to near zero in a graded fashion would be supplemented by a separate, guessing process at exactly zero allocation.

The presence of stimulus-invariant distributions of responses, i.e., guessing, is discordant with most modern information-processing approaches in perception and memory. Such an invariance reflects a total loss of information, and in the discrete-slot model, this loss is the result of not representing the item in WM. Total loss is not how errors come about in most models and theories. The vast majority of cognitive processing theories posit graded, noisy representations of stimuli and have no recourse to special states of total information loss. Even though these representations are noisy, they still contain some stimulus information. A very selective list of models that have no recourse to guessing states are signal detection models (Swets, 1961; Tanner & Birdsall, 1958), neural network models (e.g., J. L. McClelland, 1993; J. M. McClelland & Rumelhart, 1988), diffusion models (e.g., Ratcliff, 1979; Ratcliff & Rouder, 1998), vector models of stimulus representation in memory (e.g., Hintzman, 1988; Gillund & Shiffrin, 1984; Murdock, 1993; Shiffrin & Steyvers, 1997), models with Bayesian updating (e.g., Ma & Huang, 2009; Sims et al., 2012), and dual-process theories of memory (e.g., Wixted & Mickes, 2010, Yonelinas, 1994). The presence of guessing—a discrete state of total information loss corresponding to responses that are stimulus invariant—would be a surprising result unanticipated by some of the more successful models in cognition.

The goal of this paper is to assess whether there is guessing in a common working memory paradigm. The task is complicated because leading models with and without recourse to guessing states tend to do a good job of mimicking one another in common paradigms. Current attempts to assess whether one type of model holds relative to another have been heavily vested in specific model assumptions as well as mired in assumptions about the range of flexibility of models (see, for example, van den Berg, Awh,

& Ma, 2014). Our approach is to improve the discriminability of the standard paradigm and data representation so the results are by-and-large obvious from visual inspection. In the next section, we provide a simple modification of a common WM paradigm that yields this discriminability. We then present the results of 4 experiments that span 121 participants. For each experiment, the evidence for guessing may be seen by visual inspection of the appropriate plots. To provide converging evidence, we fit a series of formal distributed-resources models without guessing and discrete-slot models with guessing. This more formal analysis indeed provides a second line of evidence for guessing in WM. The consequences for theories of WM specifically and for cognition more broadly are provided in the General Discussion.

Paradigm, Data Representation, and Predictions

The paradigm we use to explore whether there is guessing is a variant of the production paradigm from Zhang and Luck (2008). Illustrations of the stimuli and the sequence of events that define a trial are provided in Figure 1. An *item* in our paradigm was a simplified jewelry ring (Figure 1A). The band was a larger open circle, and the gem was a filled smaller circle. The key attribute of these rings was the angular displacement of the gem, which is referred to as the *study angle* for the item. Study angle is measured with reference to the vertical, and in Figure 1A, the study angle is -45° . At study, several such rings are presented (see Figure 1B), and at test, only one-half second later, participants are presented one ring with the gem upright. Their task is to move the gem to that ring's study angle. We term the produced angular displacement as the *response angle*. The ideal response angle is the study angle, as indicated with an open circle in the figure. Performance in this task declines with the number of items in the study display, and this number is termed the *set size* throughout.

In most of the previous experiments with this paradigm (e.g., Anderson, Vogel, &

Awh, 2011; Bays, Catalao, & Husain, 2009; Zhang & Luck, 2008, 2009), the study angle was varied uniformly across all angles on the circle. In the current experiments, the study angles were restricted to a much smaller range. For example, in Experiment 1, the study angle was varied uniformly across the top third of the circle (from -60° to 60°) rather than across the whole circle. This restriction proves important for reasons that will become clear subsequently.

Figure 2 shows the data representation and predictions for basic versions of the discrete-slot and distributed-resource models. The study angle for the test item is presented on the x-axis; the response angle to the test item is presented on the y-axis. Perfect performance where the response angle exactly matches the study angle is indicated by the positive diagonal. Of course participants' data deviates from the diagonal and different models describe different patterns of deviation. Figure 2A shows the predictions for the distributed-resource model from Bays & Husain (2008). Here, response angles are perturbed from the positive diagonal due to limited resources. The degree of spread is an inverse function of the amount of resources. As the set size increases, the amount of resources per item decreases, and, consequently, the spread increases. The overall geometry of the pattern is that responses follow a series of diagonal bands that increase in width with set size.

Figure 2B-C show the predictions for simple versions of the discrete-slot model. Here, performance is a mixture of responses from items in WM and those not in WM. Responses to items in WM reflect the study angle fairly accurately. These responses correspond to the narrow band near the diagonal. For a set size of 1 item (left panel of Figure 2B), all studied items are assumed to be in memory, and the pattern is a narrow diagonal. If the set size is higher than an individual's capacity, then some of the responses are guesses. The distribution of these guesses do not vary with the study angle. Figure 2B shows the case where guesses are normally distributed with a center at 0 degrees. Because

the distribution cannot vary with study angle, the geometry of the resulting pattern is a horizontal band. As the set size increases, there is a greater proportion of guesses and a smaller proportion of responses from WM. This behavior is seen as an increasing proportion of responses in horizontal bands with increasing set size.

Figure 2C shows the discrete-slots model for a different guessing distribution. Here, when participants guess, they distribute their responses around stereotypical angles for leftward and rightward displacements rather than around the vertical center. Guessing is still indicated by a diagnostic horizontal banding structure. The bimodal guessing pattern in Figure 2C will characterize our data, and we suspect its presence is a result of the restricted range modification with a center-point at the vertical orientation.

We do not prejudge the distribution of guessing responses should guessing indeed occur. This distribution may be uniform (as instantiated by Zhang & Luck in their full-range experiments), normal, or bimodal. The more critical question for us is whether there is guessing. Guessing, should it occur, entails the following constraints on data: 1. Guesses are represented as a horizontal banding structure, and 2. This structure cannot vary with set size or with other manipulations. For instance, if the horizontal band structure is bimodal for three items with bands at $\pm 30^\circ$, then this same structure must be present for six items as well. The evaluation of these constraints is relatively straightforward within our data representation where response angle is plotted as a function of study angle and with the restricted-range version of the task.

Previous work with full-range stimuli has not leveraged these constraints. Typically, data are collapsed across study angle and the distribution of errors is plotted. These distributions may then be modeled as a two-component mixture, supporting discrete slots, or as continuous mixture, supporting distributed resources. In the current data representation, in contrast, the constraints from guessing are visually highlighted as different geometrical structures and may be assessed by inspection without recourse to

detailed parametric assumptions. The restricted range of study angles turns out to be useful because as will be shown, it encourages participants to guess in a stereotypical, easily diagnosable pattern.

Although diagonal and horizontal bands are key diagnostic patterns anticipated by discrete-state and distributed-resource models, we observed one additional pattern. The pattern, termed *coarse encoding*, is as far as we know novel in this domain. The pattern is shown in Figure 2D, and is characterized as a step function. For leftward study angles, responses are made at a stereotypical leftward angle, say around -30° . For rightward study angles, the responses are made at a stereotypical rightward angle, say around $+30^\circ$. Coarse encoding is different from guessing because the participant stores the correct direction of the study angle. Missing, however, is magnitude information. We refer to representations that give rise to responses on the diagonal as *fine encodings* because both direction and magnitude were encoded, albeit imperfectly. Figure 2D shows a mixture of finely and coarsely encoded items.

In summary, there are three main patterns that are diagnostic of working-memory processes in this paradigm. The diagonal band is a signature of finely encoded representations, and a widening with set size is a signature of distributed resources. Horizontal bands that extend across all study angles is a signature of complete information loss, and the guessing implied by these bands are naturally compatible with a discrete-slot model. Finally, horizontal bands that extend on half the range are a signature of coarsely encoded representation, a pattern not anticipated by either class of models.

To foreshadow, the patterns from the following 4 experiments are better described as a mixture of a diagonal and horizontal bands, and by inspection the presence of guessing is obvious. The horizontal bands, however, are heavier for the side of the studied angle indicating some coarse encoding. To better understand these patterns, we analyzed a number of distributed-resources and discrete-slot models including those with

course-encoding components. We first present Experiment 1, and use the results to guide and motivate model development.

Experiment 1

Experiment 1 is designed to assess the nature of visual WM with the paradigm presented in Figure 1.

Method

Participants. Twenty-four students (13 female and 11 male) at the University of Missouri completed the experiment as part of an Introduction to Psychology course requirement.

Design & Stimuli. The independent variables of this study were set size (either 1, 3, or 6 items), and study angle (finely graded between $\pm 60^\circ$), both of which were manipulated in a within-list manner. Set sizes were evenly distributed, with 1, 3, and 6 items at study appearing one-third of the time each. Study angle on each trial was drawn randomly from a discrete uniform from -60° to $+60^\circ$. To avoid having an exact upright position, the value of 0° was excluded from the set of possible study angles.

Stimuli consisted of colored items as shown in Figure 1B. There were eight possible locations, which were spaced evenly on the circumference of circle. Each of these eight locations corresponded with a specific color, for example, the Northeast location corresponded to the color grey. If an item was displayed at a certain location, it was displayed in the corresponding color. The colors were red, green, blue, yellow, magenta, cyan, grey, and light blue for locations. This correspondence serves as a redundant cue which may reduce the probability that the participant mistakenly recalls the wrong item.

The experiment was run under DOS on PCs using the Allegra library for real-time control of the display.

Procedure. Trials began with 500 ms of fixation. The study array was then presented for 1000 ms. Following study, a fixation was presented for 500 ms, followed immediately by the test item that remained visible until a response was made. The test item was shown with the gem at 0° . Participants were asked to produce the study angle by moving the gem. To do so, they successively pressed the left and right arrow keys, which moved the gem a small increment counterclockwise and clockwise, respectively. Participants hit the space bar to indicate that they were satisfied with their response, and response angles were restricted between -75° and $+75^\circ$. Sessions began with a practice block, and was followed by 6 experimental blocks of 60 trials each.

A total of 360 trials were run per participant. Of these, the first 30 were considered practice and not analyzed. The responses to the remaining 330 comprised the data for analysis.

Results from Inspection

The resulting data are shown in Figure 3A. Each point comes from a single participant responding on a single trial. The main structure of these plots is the presence of both a diagonal and of horizontal bands. These horizontal bands, indicate a response mode that is invariant to study angle and are signatures of uninformed guessing. The specific pattern of guessing is most evident for six items, where presumably guessing is most prevalent. Accordingly, when people guess, they seemingly first choose a side, left or right, with equal probability. Then, given the side, they make a stereotypical response, that is one centered around $\pm 30^\circ$, which is the mean of of the negative and positive study angles, respectively.

The graph in Figure 3A shows aggregate trends as the points are pooled across different participants. There is always a danger that these trends may reflect the variability across individuals more than an underlying cognitive signature of each

individual's processing (Estes, 1956). Figure 3B provides individual histograms for extreme positive study angles ($\geq 50^\circ$) for six items. Each individual has many responses between 25° and 50° , which is expected given the positive study angle. What is diagnostic is the smaller mode around -30° , which corresponds to the lower horizontal guessing band in Figure 3A. This mode appears for a large majority of the participants, showing that it is not from aggregating across a few participants. The robust presence of horizontal bands, a signature of uninformed guessing, has not been previously documented.

Our results are similar to Zhang and Luck (2008) in that we find that response angle is a mixture of a mnemonic component and a guessing component. We find the guessing component is bimodal, that is, it itself is a mixture of stereotypical leftward and rightward angles. Zhang and Luck, in contrast, found uniform guessing. This difference may reflect the fact that we used a restricted range of study angles (from -60° to $+60^\circ$) whereas Zhang and Luck did not restrict their study angles. In restriction of the range, the upright orientation (0°) serves as the midpoint. Participants may infer stereotypical left and right positions, which in the current design are at $\pm 30^\circ$, and center guess responses on these values. In summary, the unanticipated benefit of the restricted range in Experiment 1 is that participants appear to have a bimodal guessing pattern that is distinct and may be easily confirmed from inspection. This pattern is replicated in subsequent experiments.

Unfortunately, the full pattern of results is difficult to reconcile with basic versions of either distributed-resources or discrete-slots models. The distributed-resource model is not compatible with the horizontal bands, which indicate the presence of uninformed guessing. The discrete-slots model is not compatible with the increase of width of the diagonal mode. This increase in width indicates that the judgment from memory becomes less precise with increasing set size. Yet, if slots work independently with one item per slot, there should be no increase. In contrast, the decrease in precision is naturally accounted for by the distributed-resources model. In the next section, we formalize these

basic models and propose a number of generalizations to better understand the constraints in the data.

Formal Model of Working-Memory Judgments

Throughout this paper we will use comparisons among formal models to supplement findings. In this section, we develop the models, in the next section apply them to Experiment 1, and thereafter we apply them to subsequent experiments. The data representation is straightforward. Each trial is described by a study angle, denoted x , a response angle, denoted y , and a set size, denoted n . The following models are defined on these variables:

Distributed-Resource Models

We fit a total of seven resources models, all of which were variants on the following base-resource model.

Model \mathcal{R}_1 : **Base Resources Model**. The main idea of a resource model is that responses angles reflects study angles perturbed by noise, and that the noise increases with the set size. The model may be given as

$$y \sim \text{Normal}(x, \sigma_n^2), \quad (1)$$

indicating that response angle is normally distributed, centered at the study angle, and has a variance σ_n^2 . The variances are subscripted by n indicating a separate variance for each set size n . For Experiment 1, there are three set sizes, 1, 3 and 6, and, consequently, there are three variance parameters, σ_1^2 , σ_3^2 and σ_6^2 . These variances are parameters, and must be estimated from the data. Because resources decline with set size, it is expected that $\sigma_6 \geq \sigma_3 \geq \sigma_1$.

The remaining resource models are all variations of this base model:

Model \mathcal{R}_2 : **Power Law Resources Model**. In the base model, there is a different variance for each set size. Such a model is quite flexible as there is no specified relationship between variance and set size. In their original formulation, Bays and Husain (2008) did specify a relationship, they posited that variances increased with set size as a power law. This specification is given by the restriction that

$$\sigma_n = \sigma_0 n^\lambda. \quad (2)$$

The variance for each set size is described with two parameters, σ_0 , the variance for one item, and λ , which describes the amount of increase in variance for additional items.

Model \mathcal{R}_3 : **Mistaken-Items Resources Model**. Bays et al. (2009) constructed an elegant and innovative alternative explanation of guessing-like behavior in these production tasks. These authors assumed that the resources model held, but, on occasion, the participant mistakenly recalled the wrong item. In this case, the response may appear to be a guess, though it reflects perhaps good mnemonic performance to an item that was not tested. Moreover, since study angles were uniformly distributed in Zhang and Luck's studies, these apparent guessing responses would be uniformly distributed as well. In Experiment 1, we uniformly distributed the study angle across the restricted range, and would expect that if individual's responded to the wrong item, their responses would be uniform across this range as well. We generalize Model \mathcal{R}_1 by allowing the participant respond to the target item with probability π . When the participant responds to the wrong item, she or he responds to each of the remaining $n_i - 1$ wrong items with equal probability, $(1 - \pi)/(n_i - 1)$. In Bays, Catalao, and Husain, the probability of correctly responding to the target depended on set size, which is denoted here as π_{n_i} for the i th trial. The full specification of model is provided in the Appendix, and the model has a total of 5 parameters for Experiment 1: σ_1 , σ_3 , σ_6 , the variances as before, and π_3 , and π_6 , the probability of selecting the correct target item from memory. We assumed that

$\pi_1 = 1$ because with one item there is no probability of drawing the wrong item.

Model \mathcal{R}_4 : **Mistaken-Items + Power-Law Resources Model**. This model is the same as \mathcal{R}_3 with the power-law restriction (Equation 2) describing how variances change with set sizes. It has 4 parameters: σ_0 , λ , π_3 and π_6 .

Models \mathcal{R}_5 & \mathcal{R}_6 ,: **Mistaken-Neighbors Resources Models**. The Bays et al. notion that participants may occasionally respond to a wrong item is *a priori* appealing. We wonder about which wrong item they respond to. In the Bays et al. specification, all non-tested studied items were equally likely to be confused with the target. We wondered, however, if participants might be likely to confuse adjacent items with the target rather than spatially-distant ones. In Figure 1B, for example, the dark red item may be confused for the neighboring light red or purple one, but not for the distant black one. Evidence for this graded representation of location in working memory tasks comes from Vul & Rich (2010) who showed that location confusions diminish with distance. Model \mathcal{R}_5 and \mathcal{R}_6 are the analogs of \mathcal{R}_3 and \mathcal{R}_4 with this modification. The specification is given in the Appendix.

Model \mathcal{R}_7 : **Trial-by-Trial-Variability Resources Model**. van den Berg et al. (2012) provided another innovative and elegant post-hoc explanation of guessing behavior in Zhang and Luck’s (2008) data. These authors explore the effects of trial-by-trial variability in resources. The notion is that due to momentary fluctuations of attention, the amount of total resources available to the participant may vary from trial to trial, even for displays with the same set sizes. We follow the specification in van den Berg et al. (2012), and for the current design with restricted ranges, the resulting model on y is

$$y \sim T(x, \beta\alpha_n^{1/2}; 2\alpha_n), \quad (3)$$

where T is a three-parameter T distribution centered at x with scale $\beta\alpha_n^{1/2}$, and degrees of freedom $2\alpha_n$. The derivation for this case is provided in the Appendix, and the free

parameters are β , which describes how variable the resources are from trial-to-trial, and α_1 , α_3 , and α_6 , which describes how resources diminish with increasing set size.

It is conceivable to specify distributed-resource models that have guessing states. For example, resources may be distributed to a subset of the to-be-remembered items, and to-be-remembered items outside this subset may receive no resources whatsoever (Sims et al., 2012; van den Berg, Awh, & Ma, 2014). The responses to the items that receive no resources are guesses, and the distribution of these responses do not depend on study angle and account for the horizontal banding. Yet, we do not pursue models of this type. We worry about the parsimony and falsifiability of such a model. If resources can be all-or-none for some items, then it seems that there always exists a resource account for all data, and the concept loses theoretical power.

Discrete-Slots Models

Model \mathcal{S}_1 : The Base Discrete-Slot Model. The basic discrete-slot model derives from the specification of item limits by Cowan (2001) and subsequently developed for production tasks by Zhang & Luck (2008). The key concept is that an item is either in memory, in which case the response angle is centered on the study angle, or not, in which case, the response angle comes from a guessing distribution. Responses are a mixture from these two states. Let $y \sim \text{Mix}(A, B, p)$ denote that y is distributed as a mixture of distribution A and distribution B with A occurring with probability p and B occurring with probability $1 - p$. With this notation, the discrete-state model is

$$y \sim \text{Mix}(M, G, \pi_n), \tag{4}$$

where M denotes responses from memory, G denotes responses from guessing, and π denotes the probability that a response is from memory. This probability is a function of set size, and consequently π is subscripted by n . The probability depends on capacity, or

number of slots denoted k , as follows:

$$\pi_n = \begin{cases} \frac{k}{n} & n \geq k, \\ 1 & n < k. \end{cases} \quad (5)$$

In the base version, set size only affects the probability of an item being in memory. Set size does not affect the distribution of responses from guessing, G , nor the distribution of responses from memory, M . Such lack of effects are called *conditional independence* by Province & Rouder (2012). We place a normal on M , e.g.,

$$M \sim \text{Normal}(x, \sigma). \quad (6)$$

After observing the results of Experiment 1, we decided to place a bimodal distribution on G , which itself can be represented as a mixture of guessing rightward, denoted g_+ and guessing leftward, denoted g_- :

$$\begin{aligned} G &\sim \text{Mix}(g_+, g_-, 1/2) \\ g_+ &\sim \text{Normal}(\mu_g, \sigma_g), \\ g_- &\sim \text{Normal}(-\mu_g, \sigma_g), \end{aligned} \quad (7)$$

where $\pm\mu_g$ is the stereotypical rightward and leftward angle, respectively, and σ_g^2 is the variance of guessing around this stereotypical angle. The model contains 4 parameters: k , σ , α , and σ_g .

Model \mathcal{S}_2 : Free Variance Discrete-Slots Model. One of the properties of the basic discrete-slots model is that the standard deviation of the memory component, σ , is constant across set size. In the data representation of Figure 2B-C, the diagonal component should have width that does not depend on set size. Yet, it is clear from inspection that there is a systematic trend with wider diagonal bands for increasing set sizes. Zhang and Luck (2008) found this trend as well.

We constructed a generalization of the basic model by allowing the standard deviation of the memory component to vary across set size:

$$y \sim \text{Mix}(M_n, G, \pi_n), \quad (8)$$

$$M_n \sim \text{Normal}(x, \sigma_n) \quad (9)$$

where σ_n describes the standard deviation from memory for given set size. Although this model maintains the parsimony of an item limit, the generalization violates the parsimony of conditional independence as the distribution of the memory component depends on set size.

Model \mathcal{S}_3 : Slots Averaging Model. Zhang and Luck (2009) devised an elegant slots-averaging approach to account for the increased variability in the memory component with set size. When the set size is less than capacity, an item may take up more than one slot. Zhang and Luck assumed that if an item was in multiple slots, then the retrieved angular value was the average of the value in each slot. This average is a more precise estimate than the estimate from any slot, hence the variability of the response angle for items in multiple slots should be less than those stored in a single slot, and the gain in precision from slots averaging is a function of $1/\sqrt{m}$, where m is the number of slots taken by the item. In Model \mathcal{S}_3 ,

$$\sigma_n = \frac{\sigma_0}{\max(1, \sqrt{k/n})}, \quad (10)$$

where σ_0 is a free parameter.

Models with Coarse Encoding

One subtle property of the pattern in Figure 3A is that there is a lot of weight on the horizontal bands for leftward response angles when the study angle is leftward, and for rightward response angles when the studied angle is rightward. This pattern, which we consider indicative of coarse encoding, is even more salient in subsequent experiments. We

constructed a set of coarse-encoding models to account for this phenomenon. On some proportion of trials, the participant encodes the appropriate side, left or right, rather than the study angle.

One rationale for coarse encoding is that it is an adaptation to time pressure. On some proportion of the time, participants may encode only direction to speed processing (cf, Zhang & Luck, 2011). We expect the probability of coarse encoding to increase with set size, and that coarse encoding would not occur with one item as there is ample time to finely encode it. These coarse-encoding models were constructed after viewing the data in Experiment 1, and are treated here as exploratory. As it turns out, the coarse-encoding models are relatively successful, and this success is placed in context in subsequent discussions.

Model \mathcal{CR} : Coarse Encoding With Distributed Resources. We took the base distributed resource model, \mathcal{R}_1 and added a coarse-encoding component as follows. Each item that is present in memory has some probability γ of being finely encoded and complement probability $1 - \gamma$ of being coarsely encoded. We assume that γ is dependent on set size, and the fine encoding rate on any trial is given as γ_n . The resultant model is a mixture:

$$y \sim \text{Mix}(M_n, C, \gamma_n), \quad (11)$$

with $M_n \sim \text{Normal}(x, \sigma_n)$ as before and C , the coarse encoding component being

$$C \sim \text{Normal}(s_x \mu_g, \sigma_g) \quad (12)$$

where s_x is the sign of the study angle x (e.g., $s(x) = x/|x|$). There are seven parameters: $\gamma_3, \gamma_6, \sigma_1, \sigma_3, \sigma_6, \mu_g$ and σ_g .

Model \mathcal{CS}_1 : Coarse Encoding With Discrete Slots. Coarse encoding may be implemented in a discrete-state model by allowing a set-specific probability of coarse and

fine encoding. The model is

$$y \text{ Mix}(M, C, G, p_m, p_c, p_g),$$

a three-way mixture of fine encoding (with responses distributed as M in Equation 6), coarse encoding (with responses distributed as C in Equation 12), and guessing (with responses distributed as G in Equation 7). The probability of an item being in memory and finely encoded is $p_m = \pi_n \gamma_n$, the probability of an item being in memory and coarsely encoded is $p_c = \pi_n(1 - \gamma_n)$, and the probability of guessing is $p_g = (1 - \pi_n)$.

Model \mathcal{CS}_2 : **Free Variance + Coarse-Encoding Discrete-Slots Model**. This model is the generalization of \mathcal{S}_2 , and like \mathcal{S}_2 , the variance of the finely-encoded memory component depends on set size. The model is

$$y \text{ Mix}(M_n, C, G, p_m, p_c, p_g),$$

where M_n is given in Equation (9).

Model-Based Results For Experiment 1

The thirteen models provide a rich set to explore the constraints in Experiment 1. We fit all thirteen by maximizing likelihood. Maximization is fairly straightforward for the distributed-resource models and may be performed efficiently with the simplex algorithm (Nelder & Mead, 1965). It is slightly more complicated with the mixture models as the likelihood function sometimes has local as well as global maxima. To increase the probability of finding global maxima, we typically use several starting values when fitting all models.

Model Selection

We calculated both AIC and BIC model selection statistics for all participants and all models. Table 1 shows the number of participants (out of 24) for which a model was

selected as best by AIC and BIC. Overall, BIC penalizes complexity more than AIC, and we think this emphasis is appropriate. However, AIC has the advantage that it may be summed to provide a single, overall measure of the badness of fit of models.

Unfortunately, it is not principled to sum BIC this way. Coarse encoding models are favored overall by summed AIC and are favored for most individual participants (23 of 24 by AIC; 18 of 24 by BIC).

One potential problem in interpreting the goodness-of-fit values in Table 1 is a certain lack of robustness in the specification of all models: None of the models could account for the occasional response that was far from the study angle for the one-item displays without dramatic change in model parameters. One of these troublesome responses is highlighted with an arrow in the left panel of Figure 3A. In discrete-slots (and coarse encoding models), such errors come about from guessing, which only occurs if capacity is less than the set size. These errors in one-item displays necessarily imply capacity is less than 1.0 regardless of performance in the other set-size conditions. In the distributed-resource model, these errors have great influence on the variability estimates, and may obscure the relationship between variance and set size.

To address this lack of robustness, Rouder et al. (2008) introduced an additional nuisance parameter to account for occasional lapses in attention. Accordingly, on some small number of trials the participants pay no attention whatsoever, for example, they rubbed their nose or closed their eyes. The key property is that these task-unrelated lapses of attention occurred equally often across set size conditions. By including the possibility of lapses, stray errors for the one-item condition will not grossly affect the fit or parameter estimates.

To explore the robustness of the model-selection findings in Table 1 to such small lapses, we constructed alternative versions of all 13 models where a small fixed percentage of responses are generated with attention lapsed (in this case, response angle was

uniformly distributed across the range). We refit all 13 models with the possibility of lapse set to .5%, 1% and 5% of responses. Although parameter estimates and fit did vary with the probability of lapsed attention there was little change in the ordering AIC and BIC scores across the twelve models. The number of participants for which each model was selected by AIC with 5% inattention is shown in Table 1. Hence, we may conclude that the results are robust to occasional stray errors from lapses in attention.

The Coarse-Encoding, Discrete-State Account of Data

The model selection statistics endorse discrete-slot models with coarse encoding. Figure 4 shows how these models account for the observed data. The figure is divided into six rows and three columns, and the data and model predictions for set size 6 from Experiment 1 are shown in the right column. Each row shows these data and predictions for a different range of study angles as indicated with the interval marker at the top of the panels. The top panel is for study angles between -60° and -40° , the next panel in the column is for study angles between -40° and -20° , and so on. In each panel the blue histogram shows the data. The red shaded density show the theoretical responses from fine encoding into a discrete slot—the amount of area shows the probability of fine encoding, and the mean tracks the range of study angles. The green shaded densities show the combined responses from coarse encoding and guessing, and the centers do not change with study angle. Notice that the leftward component is higher than the rightward for leftward study angles and the reverse holds for rightward study angles. This difference is the effect of coarse encoding. The sum of fine encoding, coarse encoding, and guessing are shown as the dashed line, and it forms the prediction for data. As can be seen, at this aggregate level, the model does a good job of accounting for trends.

Model Selection Through Parameter Estimation

We performed model selection through AIC and BIC measures. Bays, Catalao, and Husain (2009) offer an alternative based on parameter estimation. They estimated the role of guessing in an omnibus model in which responses may come from one of three possible mechanisms: responses may come from working memory, in which case the response angle is closely related to the stimulus angle; responses may come from the working memory of studied nontarget in which case the response is closely related to the study angle of a nontarget; or responses may come from a guessing process. Bays et al. estimated the probability that a response was from one of these three mechanisms. They considered the resource model successful if guessing-like behavior was not from guesses but instead from responses to nontargets. We adopt Bays et al.'s logic here to estimate just how much guessing cannot be attributed to the identification of nontargets. Model \mathcal{R}_8 , specified in the Appendix, is a straightforward generalization of \mathcal{R}_3 that has a memory process to the correct target, a memory process to a nontarget, and a guessing process. The weight or probability of each of these processes is estimated.

The critical question is whether these guessing weight parameters are zero or greater than zero. Figure 5A shows the estimates of these guessing parameters for all participants, and clearly there is overwhelming evidence for guessing. We formalized this observation by performing a nested likelihood ratio test between Model \mathcal{R}_3 and Model \mathcal{R}_8 , which differ only in the presence or absence of guessing. Model \mathcal{R}_3 may be rejected for all participants (at $\alpha = .05$) indicating that guessing is needed even when responses to nontargets are possible. Our results differ from Bays et al. who found that guessing behavior could be accounted for by confusing nontargets for targets. We think this difference may reflect several factors, but perhaps the most important is that guessing in this task is bimodal. Bimodal guessing cannot be accounted for by target confusion because the target confusion results in responses that are distributed similar to the

distribution of study angles, which in this case is uniform. We note that in our paradigm that confusion is controlled by presenting items in constant, well-specified locations and having a consistent mapping between color and location that does not vary across trials. These elements were not present in Bays et al. and may have contributed to possible target-nontarget confusions in their paradigm.

Estimates of Latent Processes

The preferred explanations for the data in Experiment 1 are coarse encoding models. Because they fit well, these models may be used to measure posited latent quantities such as the capacity of WM (number of slots), the probability of coarse encoding, and the probability of pure guessing. To provide for robust estimation of parameters, we added an attention-lapse parameter as recommended by Rouder, Morey, et al. (2008). The new measurement model, denoted \mathcal{CS}_3 , is proper generalization of \mathcal{CS}_2 in which there is an additional parameter ω , the probability of an attention lapse. When attention lapsed, participants guessed with the same bimodal pattern when items were not in memory. The model is given in the Appendix.

Median estimates of participant-level parameters are shown in Table 2. Median participant capacity was 2.7, which is consistent with production estimates from Anderson et al. (2011). Lapses in attention were indeed rare (.02). Figure 5B shows estimates of guessing across participants and set sizes. As can be seen, guessing is negligible for one item (.02) but substantial for six (.55), and this pattern held across participants. Figure 5C shows the degree of coarse encoding. From the figure it is clear that coarse encoding is a robust phenomena that occurs for all participants.

Interim Summary

The inspection of trends in Figure 3 supplemented by model-based analysis yield a number of novel results:

1. There is evidence that on some proportion of the trials participants guess without information. The signature of guessing is horizontal bands that span the range of study angles, and these are clearly visible in Figure 3A. Estimates from the models revealed that participants guessed on about half the trials in the six-item condition.

2. When participants guessed, they did so with a bimodal pattern that was characterized by stereotypical responses. This pattern is evidenced by the presence of two distinct horizontal bands in Figure 3A as well as by the bimodalities in Figure 4.

3. The results indicate the presence of coarse encoding, where participants correctly encoded the side (or sign) of the study angle, but produced a stereotypical magnitude. The evidence for coarse encoding comes from the excessive density of correct-direction responses at the stereotypical locations (see Figure 3 and Figure 7) and from the success of the coarse encoding models (\mathcal{CS}_1 and \mathcal{CS}_2). Coarse encoding was estimated with Model \mathcal{CS}_3 and occurred on .42 and .59 of the trials where target is encoded for set sizes 3 and 6, respectively (see Table 2). We speculate that coarse encoding is a strategy participants use when attempting to encode many items in a short interval.

The need for coarse encoding to account for data in a discrete-slots framework is not necessarily appealing from a theoretical perspective. Coarse encoding is a sizable and complicating revision that limits, to some extent, the parsimony of the slots model. Nonetheless, the data for it seems quite evidentiary. Further commentary on these limitations and outstanding theoretical problems are presented in the General Discussion.

In the following set of experiments, we further assess their presence and explore the role of guessing in processing. Experiment 2 is a replication of Experiment 1 with a few small changes in the procedure. Experiment 3 is an exploration of guessing behavior. In it, participants were forced to guess on some trials, and the main question was whether guesses on these trials match in distribution guesses derived from the discrete-slots WM models. In Experiment 4, we manipulate base rates of certain study angles and assess

whether this manipulation selectively influences responses when guessing. To foreshadow, these experiments confirm the above summary points about guessing, coarse encoding, and the inapplicability of resources.

Experiment 2

Experiment 2 is nearly a replication of Experiment 1. The main difference is how responses were collected from participants. In Experiment 1, participants used multiple key presses on a keyboard to finely place the jewel of the test ring. In this study, participants used a mouse to place the jewel. The primary goal was to assess whether the novel bimodal guessing pattern and coarse-encoding phenomena replicated rather than to explore the effects of the response-collection procedure.

Method

Participants. Twenty-six students (16 female and 10 male) at the University of Missouri completed the experiment as part of an Introduction to Psychology course requirement.

Design, Stimuli, and Procedure. The design, stimuli, and procedure followed that of Experiment 1 with the following exceptions: (1.) Study angles were sampled from a discrete uniform distribution on the interval -60° to 60° in increments of 1° . Unlike in Experiment 1, it was possible for items to be oriented exactly upwards (0°), and this orientation occurred with probability $\frac{1}{121}$. (2.) The experiment was run on MAC OSX computers using the Psychophysical Toolbox under Octave. The stimulus colors of orange and violet were used instead of magenta and light blue. (3.) Responses were collected by mouse instead of by button presses. When the test item was presented, the jewel was placed in the center of the ring rather than on the circumference. The participant then moved the jewel with the mouse toward the ring. When the jewel was near the ring, it

locked onto the ring, and all other adjustments were adjustments of the angle on the ring. Participants found this procedure simple and intuitive in practice. Participants clicked on the mouse button to indicate that they were finished adjusting the jewel and the final displacement was recorded as the response angle. (4.) Participants could place the jewel anywhere on the circumference of the ring, and this range is in contrast to Expt. 1 where response angles were limited to $(-75^\circ, 75^\circ)$. The experiment is still a restricted-range experiment as study angles were restricted to the upper third of the ring. (5.)

Trial-by-trial feedback was provided. After response, participants were shown their response and the original study angle. Simultaneously, audio feedback was provided, and this feedback was modeled after Nintendo video game noises. If the response was very close to the studied angle (within 10°), then the participant heard a pleasant ascending sequence ($C_1 - E - G - C_2$, where C_1 is Middle C and C_2 is one-octave higher). If the response was somewhat close to the studied angle (between 10° and 30°), then the participant heard a pleasant descending sequence ($G - E - C$). Otherwise the participant heard a less-pleasant descending sequence ($C_1 - D\# - D - C_0$, where C_0 is an octave below Middle C). **6.** The seven experimental blocks of 60 trials each were preceded by a small block of twelve practice trials.

Results & Discussion

The aggregated results for Experiment 2 can be seen in Figure 6, and the pattern is similar to that from Experiment 1. Although there is a horizontal component in the pattern, it is not as obvious as in Experiment 1. To help interpret the pattern, we performed the same model-based set of analyses for Experiment 2 as we did for Experiment 1, and the resulting AIC and BIC statistics are reported in Table 1. These results are quite similar to Experiment 1, and in particular the selected models were slot models with coarse encoding. The account of aggregated data in Figure 4 is reasonable,

and there is evidence for guessing and for coarse encoding from parameter estimation (see Figure 5).

Experiment 3

Experiments 1 and 2 indicate the presence of guessing, a response component that is invariant to study angle. To further assess the role of guessing, we introduce a new condition, called the *false-probe condition*, in which stimulus-unrelated guessing is assured rather than inferred. In this condition, no stimulus is actually studied so the response must be a guess. On a false probe trial, several rings were presented, but the test item is presented at a position (and in a color) that was not present at study. Consider Figure 1B. Here the test item was indeed studied, but suppose the test item occurred at the West or 9:00 position instead. There is no study item at that position, and, consequently, the response must be a guess. Performance on false-probe trials may be contrasted to *true-probe* trials where tested items were indeed studied (as in Experiments 1 and 2). We use the models to infer the distribution of guessing on true-probe trials and compare them to the pattern observed in false-probed trials. A match between responses on false-probe trials and inferred guesses on true-probe trials would provide further evidence for stimulus invariant responses, presumably from those with complete information loss from items that are not in WM.

Method

Participants. Twenty-seven students (7 female and 20 male) at the University of Missouri completed the experiment as part of an Introduction to Psychology course requirement.

Design, Stimuli, and Procedure. Experiment 3 was similar to Experiment 2; the main difference was the inclusion of false-probe trials. False probe trials occurred only on

trials with study arrays of six items, of which half had a false-probe at test. Additional minor procedural differences were: **1.** Study angles were sampled from a discrete uniform distribution on the interval -60° to 60° in increments of 1° , with the 0° position excluded. **2.** Participants could respond with angles between -75° and 75° . **3.** The seven experimental blocks of 60 trials each were preceded by a small block of twelve practice trials. **4.** The study array was presented for 500 ms with a 1000 ms blank following instead of a 1000 ms study duration with a 500 ms blank. **5.** No feedback was given to participants after they gave a response.

Results

The results for Experiment 3 can be seen in Figure 7. For true-probe items (three left panels), the response pattern for this experiment is similar to that in the previous two experiments in that the horizontal guessing bands are apparent. For false items (right panel), responses appeared to match the horizontal bands produced in true-probe item tests, suggesting that the bands on true-probe trials reflect pure guesses.

Modeling False-Probe Trials. For the discrete-slot and coarse encoding models, we assumed that participants guessed on false-probe trials with the same guessing distribution they used when items are not in memory. For distributed resource models with target confusion (Models \mathcal{R}_3 through \mathcal{R}_7), we assumed that for false probes, participants confused the probed item with one of the studied items as specified in the particular model. Resource models \mathcal{R}_1 and \mathcal{R}_2 contain no mechanism to respond to false probes. We supplemented these models with a guessing distribution, given in (7), and assumed that participants always guess on false-probe trials but never guess on true-probe trials.

Model Selection & Parameter Estimation. The relevant AIC and BIC statistics for the model-based analysis are provided in Table 1. The pattern is again fairly similar to the previous experiments, and coarse-encoding discrete-slot models outperform

competitors and well accounts for aggregate data (Figure 4). Parameter estimates for Model \mathcal{CS}_3 are provided in Table 2, and, again, there is evidence for guessing and for coarse encoding (Figure 5).

Comparison of Guessing Across False-Probe and True-Probe Trials. To directly test the proposition that the distribution of responses on false-probe trials was identical on true-probe and false probe trials, an additional coarse-encoding measurement model was constructed. Model \mathcal{CS}_3 specifies that guessing is the same for false and true-probe trials, and needed is a model where they are not necessarily the same. In a new model \mathcal{CS}_4 , specified in the Appendix, there are separate mean and variance parameters for the components in the guessing distribution inferred from true-probe trials and false-probe trials. Because Model \mathcal{CS}_3 , the model with the same guessing and false- and true-probe trials, is properly nested in Model \mathcal{CS}_4 , the equality of guessing may be tested with a likelihood ratio test (Riefer & Batchelder, 1988; Glover & Dixon, 2004; Myung, 2003). We found that, for 22 of the 27 participants (81%), we could not reject \mathcal{CS}_3 in favor of \mathcal{CS}_4 at $\alpha = .05$. Hence, there is some evidence that the restriction is reasonable. It would be far better to report a calibrated measure, such as Bayes factors (Jeffreys, 1961, Kass & Raftery, 1995, Rouder et al., 2009), but, unfortunately, the development of these measures for this class of models is quite difficult and outside the scope of this paper.

Discussion

The results with the false- and true-probe conditions conform to those in the previous experiments: (a) distributed-resource models fare poorly; (b) some responses on true-probe trials are due to guesses; and (c) there are both fine and coarse representations. Additionally, model analysis reveals the distribution of responses from false-probe trials often matched those from guessing on true-probe trials. In summary, Experiment 3 provides extended support for the previous conclusions that WM has flexibility to store

categorical and metric codes, and that when items are not in WM, they are mediated by a stimulus-unrelated guessing process.

Experiment 4a & 4b

Guessing should be sensitive to certain bias manipulations. In Experiments 4a and 4b we implemented a base rate manipulation. For one group of participants, the target was studied at a leftward orientation (negative study angles) for 80% of the trials and was studied at a rightward position (positive study angle) for the remaining 20% of trials. For a second group, these proportions were reversed. We term these conditions the *leftward prevalent* and *rightward prevalent* conditions, respectively.

According to the discrete-slot models this prevalence manipulation should affect performance responses that are mediated by guessing with responses from guessing being biased toward the more prevalent orientations. For the resources models, there is no obvious locus for the effects. Consequently, we generalized the resource models by stipulating that the response may not be centered on the veridical study angle, but may be systematically shifted to reflect the prevalence manipulation. There is a negative shift for the leftward prevalent condition and a positive shift for the rightward prevalent condition.

We ran two versions of the manipulation. In Experiment 4a, all items in the study set obeyed the prevalence manipulation. In Experiment 4b, in contrast, only the target item obeyed the prevalence manipulation, and the other nontarget items were equally likely to be leftward or rightward oriented. In both experiments, false trials were included with true-probe trials to provide a separate estimate of bias when guessing.

Method

Participants. Nineteen students (12 female and 7 male) and 25 University of Missouri students (12 female and 13 male) took part in Experiments 4a and 4b,

respectively. These students participated as part of an Introduction to Psychology course requirement.

Design, Stimuli, & Procedure. Experiment 4a and 4b were replications of Experiment 3 with an additional prevalence manipulation. For the rightward-prevalent condition, negative study angles occurred for 80% of trials. When the study angle was negative, it was uniformly distributed between -60° and -1° . Likewise, positive study angles occurred on 20% of trials, and these were uniformly distributed between 1° and 60° . The set sizes for both experiments were 2 and 7 items.

Results

The results for Experiment 4a may be seen in Figure 8, and the pattern for Experiment 4b is highly similar and omitted for brevity. The figure, however, is not that helpful for assessing whether the effect of prevalence is better captured by discrete-states or resources. The primary pattern is that there are more responses in the ranges of the more prevalent stimuli, which assuredly reflects that there are more stimuli in that range.

A more informative approach is to plot the density of responses conditional on specific study angles. Figure 9A shows these plots for Experiment 4a to targets with study angles between -40° and -30° . The lines estimate the density function, and are smoothed histograms with area normalized to be 1.0.¹ The plot is diagnostic among several different theoretically relevant possibilities. First, there could be no effect of the prevalence manipulation. In this case, the densities from the leftward- and rightward-prevalent conditions (solid and dashed lines, respectively) should overlap. They do not; in fact, they are quite different. Second, there could be a shift where responses in the leftward-prevalent condition could be shifted leftward relative to responses in the rightward-prevalent condition. Such a pattern would be compatible with a resources account, but does not occur. Third, there could be a change in guessing patterns when

guessing. For the leftward prevalent responses, there could be a higher probability of guessing with a leftward stereotypical angle and for the rightward prevalent condition there could be a higher probability of guessing with a rightward stereotypical angle. Indeed, this last pattern describes the data. For Figure 9A, both responses from memory and those from guessing contribute to the sharp mode around -40° . In the rightward-prevalent condition, there is a second mode for positive study angles, and this is affirmative evidence for a prominent guessing band for stereotypical positive angles expected in this condition. Figure 9B shows the same set of plots for leftward study angles between 30° and 40° in Experiment 4a. Here, the mirror pattern holds; there is a guessing band for stereotypical negative angles in the leftward-prevalent condition. The density plots for Experiment 4b are shown in Figure 9C-D, and they show the same pattern.

The patterns in Figure 9 are highly evidentiary; in fact, they are the proverbial smoking gun. These plots not only show mixtures, but a clear ability to selectively influence the probability of mixture components. Such lawful findings are relatively rare in cognitive psychology. The current patterns are highly constraining and serve as strong corroborating evidence for guessing. Our view is that the evidence from the plot is transparent and decisive. It is seemingly incompatible with any interpretation that does not include a major contribution of guessing. Guessing is seemingly the overwhelming cause for inaccuracies in this task.

Models. We constructed a set of resource models to account for bias from the prevalence manipulation. In these extensions, the mean of the response was not x , the study angle, but $x + \delta$, where δ was free to vary across participants, and could track the between-subject manipulation of prevalence. With just two set sizes, there is no point in constructing extensions of \mathcal{R}_{2a} , \mathcal{R}_{4a} , and \mathcal{R}_{6a} as these models are equivalent to \mathcal{R}_{1a} , \mathcal{R}_{3a} , and \mathcal{R}_{5a} , respectively. Likewise, this bias generalization was included in a distributed-resource coarse-encoding model, denoted \mathcal{CR}_a

We also constructed a set of discrete-slot models \mathcal{S}_{1a} and \mathcal{S}_{3a} to model the effect of prevalence on guessing. In the previous models, guessing was symmetric and bimodal. In the current model, there is an additional parameter, denoted η that weights the guessing mode. Parameter η is the probability of a stereotypical leftward response when guessing, and it is expected to be high in the leftward prevalent condition and low in the rightward-prevalent condition. Likewise, analogous versions of \mathcal{CS}_1 and \mathcal{CS}_2 , denoted respectively \mathcal{CS}_{1a} and \mathcal{CS}_{2a} were constructed.

Model Selection. The model selection fit statistics are shown in Table 3. These results are very similar to the previous experiments in that the resources model received no support, and for most participants both fine and coarse encoding was necessary to account for the data.

The Influence of Base Rates. In the discrete-slot and coarse encoding models, the frequency manipulation is hypothesized to selectively influence the responses while guessing (η) and not affect WM parameters such as capacity. We used Model \mathcal{CS}_{3a} , the version of \mathcal{C}_3 with a free parameter η that denotes the probability of a stereotypical leftward response when guessing.

The distribution of select parameter estimates across individuals from both experiments is shown in Figure 10. Here, the effect is clearly in the guessing parameters η for both true-probe and false probe trials. Moreover, there is no apparent differences in capacity. These trends may conveniently be assessed by a t -statistic, and we use Bayes factors rather than p -values to calibrate the evidence for comparisons (Rouder, Speckman, Sun, Morey, & Iverson, 2009). The Bayes factor is the probability of the data under one hypothesis relative to the probability of the data given another. The Bayes factors favor a large effect for the guessing parameters (true-probe trials, $t(42) = 8.096$, $B_{10} = 2.18 \times 10^7$; false-probe trials, $t(42) = 13.186$, $B_{10} = 2.86 \times 10^{13}$) and the no-effect null for the capacity

parameters ($t(42) = -0.575$, $B_{01} = 3.87$).

Discussion

The results of Experiments 4a and 4b provide additional support for the previous conclusions. The pattern of results is broadly consistent with the coarse-encoding model in which items are stored coarsely, finely, or not at all in WM. A prevalence manipulation affected guessing parameters in a reasonable manner and did not affect WM capacity. The data therefore strongly implicate guessing and are difficult to reconcile with resource models.

General Discussion

The key finding in this paper is clear and consistent evidence for guessing. There is a robust finding of stimulus-invariant responses as indicated by horizontal bands in Figures 3, 6, and 7. Moreover, this interpretation of guessing is bolstered by the concordance between the model-inferred guessing components and the performance on false-probe trials, which necessarily reflect a stimulus-independent process. The prominence of the leftward and rightward guessing bands are selectively influenced by a prevalence manipulation. We believe the most theoretically straightforward and parsimonious account of these data is to include a mechanism of complete loss where the participants have no knowledge whatsoever of the stimulus on some of the trials.

Mechanisms of complete information loss are not common in cognitive modeling, and is certainly not popular in accounts of memory. We think that pure guessing remains relatively rare because researchers do not design paradigms and assess data in ways that would reveal it. In our case, we asked participants to produce a continuous response rather than a dichotomous one, and were able to show that the distribution of these responses was a mixture of two components with one component not dependent on the stimulus variable. Such an analysis is not provided by looking at accuracy alone or

examining means. We have used a related approach to explore whether there is a guessing component in recognition memory (Province & Rouder, 2012), and find evidence for a stable guess state in this domain (see also Broder & Schutz, 2009; cf. Dube & Rotello, 2012). We suspect that these and related results carry into a number of domains, and hope that researchers find the possibility of complete information loss theoretically useful and design paradigms that have the potential to test for it.

Incompatibility with Resource Accounts

Guessing is not a characteristic of distributed-resource accounts, and the analyses point to a dramatic failure of the tested distributed resource models. There are two recently proposed modifications of the distributed resources account that account for guessing-like behavior. One, from Bays et al. (2009), is that participants confuse targets with nontargets in the study array. In our paradigm where the distribution of study angles of nontargets and targets alike is uniform, these confusions would result in uniformly distributed responses that are unrelated to the study angle of the target. Yet, we observed a bimodal guessing pattern rather than a uniform one, and this bimodality cannot be from responses to nontargets. The other modification comes from van den Berg et al. (2012) who proposed that resources varied from trial to trial even for the same set size. Guessing-like behavior reflects those trials with low resources. This variation does not distort the basic patterns in Figure 2. Instead, the distribution of perturbations from the diagonal follow a t -distribution rather than a normal. The model as implemented does not explain the observed bimodal guessing patterns. There are variable resource models with item limits (Sims et al., 2012; van den Berg et al., 2014), and such models are quite flexible and can account for guessing. Such models are similar to but more general than the discrete-slot model with free variance (Model \mathcal{S}_2). They have an additional flexibility because there is no constraint that capacity be constant.

We imagine that it may be possible to specify a resource model with criterial settings without an explicit item limit. Consider, for instance, a model with two criteria setting with an upper and lower criteria. If resources for an item are above an upper criteria, then the study angle is finely encoded, perhaps with precision a function of the amount of resources. If resources are in between the upper and lower criteria, then the side of the angle is coarsely encoded. Finally, if resources are below the lower criteria, then the response is mediated by guessing. Such a model, however, has a limited role for resources as these are a proxy for which state the item is encoded in.

Conclusions About Slot Models

The finding and analyses present substantial difficulties for discrete-slot theory as well. One issue is that accuracy for WM mediated responses (those that are not guesses) decreases with increasing set size. This decrease is unexpected because once an item is in a slot, it is memorized and should be reported independently of other items. Zhang and Luck (2008) offered a *slots-averaging* explanation where items may be represented in multiple slots to increase the precision of the response. Slots in this model take on the role of discretely distributed, limited resources. If these representations are independent and then averaged, the standard deviation of memory-driven responses should increase as \sqrt{n} , where n is the set size, as long as there are fewer items than slots. If there are more items than slots, then the standard deviation should not be affected by set size. The Zhang and Luck explanation receives support from Anderson et al. (2011) who find such a plateau, though is contradicted by the findings of Gorgoraptis, Catalao, Bays, & Husain (2011) who do not. Slots averaging strikes us as difficult to test as a mechanism in practice.

More importantly, slots theory is challenged by the presence of coarse encoding. The implication is that participants may represent items either finely or coarsely. It is reasonable to expect that if this is the case, then there is some type of trade-off where

coarsely encoded items use fewer mnemonic resources, which, again leads to a resources-like conceptualization (cf. Zhang & Luck, 2011). Moreover, it is hard to reconcile coarse encoding with slots averaging as the average of coarse and fine representations seems strained.

One of the challenges facing WM researchers is constructing parsimonious, falsifiable models that predict easy-to-observe invariances in data. The notion of an item limit serves as a testable simplifying principle. One way of clarifying the role of an item limit in theory building is asking whether it is either sufficient or necessary to account for the data.

Consider first whether an item limit is necessary to account for the data. We believe there is now fairly strong multiple points of convergence that an item limit is necessary. The evidence for these limits comes not only from a rich set of guessing invariances demonstrated here, but from the following as well: **1.** Cowan et al. (2005) and Pashler (1988) found that theoretically motivated measures of effective capacity in a change-detection task depended in an orderly manner with size of the set of to-be-remembered items. Effective capacity increased with set size in a one-to-one fashion until about four, and remained constant at this value as set size increased further. **2.** Similarly, Awh and colleagues have consistently found that the variability of responses increased with set size up to about 4 items and maintained a plateau thereafter (Awh, Barton, & Vogel, 2007; Barton, Ester, & Awh, 2009; Anderson et al., 2011). This pattern of increase for small set sizes and plateau thereafter may be seen for other behavioral and physiological measures including ERP (Vogel & Machizawa, 2004) and fMRI signals (Todd & Marois, 2005). **3.** Rouder, Morey, et al. (2008) showed that receiver operating characteristic curves from the change-detection task were straight lines with a slope of 1.0, which is the signature of a mixture of accurate performance and pure guesses. The interpretation is that the accurate component is from items in working memory while the pure guess component is from those not in working memory. Donkin, Nosofsky, Gold, &

Shiffrin (2013) show the same basic mixture finding holds when response times are modeled as a ballistic accumulator process. **4.** An item limit is also compatible with data from verbal working memory (e.g., Chen & Cowan, 2009; Cowan, Rouder, Blume, & Saults, 2012; Jarrold, Tam, Baddeley, & Harvey, 2010). Hence, it strikes us as eminently reasonable that item limits are a necessary part of any explanation. It is the convergence of these results, along with the guessing results provided here, that provide support for the necessity of an item limit.

The remaining question is whether a simple item limit is sufficient by itself to explain the extant data, or whether other limits need to be postulated. We suspect item limits are not sufficient, and there are additional limits on features and bindings. Initially, Luck & Vogel (1997) found evidence that items rather than features occupy slots when chunking or grouping is controlled. Performance in this task varied with the number of items but not with the number of features. Subsequent studies have not exactly replicated that extreme finding but they have been consistent with a model in which the number of items in working memory known in at least one feature is fixed, even though attention to multiple features decreases the number of objects for which any particular feature is known. That is, an object limit in terms of slots must be augmented by a limit in the number of independent components within each object slot (Cowan, Blume, & Saults, 2013; Hardman & Cowan, in press; Oberauer & Eichenberger, 2013). It may be that the same slot limit applies once for each feature type that is attended and again for the total number of objects constructed from the features. A related, unresolved issue is whether the binding of two features of an object naturally falls out when the same object happens to have both features in working memory (Vul & Rich, 2010) or whether some other cognitive resource must be added to hold bindings in working memory, at least when the features are perceptually separable (like color and orientation) rather than integral (like height and width) (Bae & Flombaum, 2013; Fougne & Alvarez, 2011).

There are a number of difficult-to-account for results in an item limit model. Those found here are the oft-replicated increase in variance of the WM-mediated responses for small set sizes and the novel presence of coarse encoding. Other difficulties include the effects of chunking (Cowan et al., 2012; Miller, 1956) and other higher-order regularities in the stimulus field (Brady & Tenenbaum, 2013; Chong & Treisman, 2005; Jiang, Chun, & Olsen, 2004). Finally, there are strategic factors that come into play when one steps outside of our narrowly-defined paradigm, such as overlooking rare objects (Wolfe, Horowitz, & Kenner, 2005). The challenge going forward is to establish paradigms and models which allow us to draw constraints on the relationship between these limits. Indeed, while the field is vibrant and evolving, there is still much work to be done.

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Appendix: Additional Model Specification

Specification of Model \mathcal{R}_3 : Mistaken-Item Resources Model

The specification of models for this task is made most concisely as a density function. Let $f(y)$ denote the density of the distribution of response angles at value y , and let $\phi(z)$ denote the density of a standard normal at z . Then,

$$f(y) = \pi_n \phi\left(\frac{y-x}{\sigma_n}\right) + \frac{1-\pi_n}{n-1} \sum_{j=1}^{n-1} \phi\left(\frac{y-x_j^*}{\sigma_n}\right), \quad (13)$$

where x_j^* are the angles of the $n-1$ distractor items.

Specification of Model \mathcal{R}_5 : Mistaken-Neighbor Resources Model

In this model, the participant occasionally confuses a target with a nearest neighbor. We used a version of Luce's (1959) choice rule to model whether a neighboring item was mistaken. We assumed that targets had strength ν , and if a neighbor was presented, it had strength 1.0. The probability of correctly retrieving the target is the target strength (ν) divided by sum of the strengths across the target and neighbors, if present. Let \mathcal{I}^+ and \mathcal{I}^- be indicator functions that are 1 if there is neighboring item one position clockwise and counterclockwise to the target, respectively, and zero otherwise. The probability of correctly recalling the target is $\pi = \nu/(\nu + \mathcal{I}^+ + \mathcal{I}^-)$; the probability of mistakenly recalling the clockwise and counterclockwise neighbors are $\pi^+ = \mathcal{I}^+/(\nu + \mathcal{I}^+ + \mathcal{I}^-)$ and $\pi^- = \mathcal{I}^-/(\nu + \mathcal{I}^+ + \mathcal{I}^-)$, respectively. The density function for the model is

$$f(y) = \pi \phi\left(\frac{y-x}{\sigma_n}\right) + \pi^+ \phi\left(\frac{y-x^+}{\sigma_n}\right) + \pi^- \phi\left(\frac{y-x^-}{\sigma_n}\right), \quad (14)$$

where x^+ and x^- are the angles at the nearest-neighbor items should they be present.

Derivation of Model \mathcal{R}_7 : Trial-by-Trial-Variability Resources Model

We modified the van den Berg et al. (2012) for the restricted-range stimuli as follows. For a fixed variance σ^2 , we start with base resource model for fixed variability:

$$f(y) = \phi\left(\frac{y-x}{\sigma_n}\right).$$

van den Berg et al. (2012) specify that the precision on each trial, the reciprocal of variance, follows a gamma distribution. This specification may be implemented by placing an inverse-gamma distribution on variance (Johnson, Kotz, & Balakrishnan, 1994). There are two parameters of the inverse gamma distribution, the shape (denoted α) and the scale (denoted β), and van den Berg et al. (2012) specify that set size affects the shape but not the scale. The density of responses may be obtained by marginalizing variance over an inverse-gamma distribution:

$$f(y) = \int_{\sigma^2} \phi\left(\frac{y-x}{\sqrt{\sigma^2}}\right) f_{ig}(\sigma^2; \alpha_n, \beta) d\sigma^2,$$

where f_{ig} is the density function of an inverse gamma distribution.² Fortunately, this integral can be expressed in closed-form. Through algebraic rearrangement, it may be shown that

$$f(y) = t\left(\frac{y-x}{(\beta\alpha_n)^{1/2}}; 2\alpha_n\right). \quad (15)$$

where $t(x; \nu)$ is the density of a t distribution, evaluated at x , with ν degrees of freedom.

Specification of Model \mathcal{R}_8 : Mistaken-Item Resources Model + Guessing

The probability density function of responses is

$$f(y_i) = \pi_n \phi\left(\frac{y-x}{\sigma_n}\right) + \gamma_n g(y; \mu_g, \sigma_g) + \frac{1 - \pi_n - \gamma_n}{n-1} \sum_{j=1}^{n-1} \phi\left(\frac{y-x_j^*}{\sigma_n}\right), \quad (16)$$

where g is the density when guessing and is given by

$$g(y; \mu_g, \sigma_g) = \frac{1}{2} \phi\left(\frac{y-\mu_g}{\sigma_g}\right) + \frac{1}{2} \phi\left(\frac{y+\mu_g}{\sigma_g}\right) \quad (17)$$

Specification of Model CS₃: Discrete-State Coarse Encoding + Attention Lapses

The probability density function of responses is

$$f(y) = (1 - \omega)\pi_n \left[\gamma_n \phi\left(\frac{y - x}{\sigma_n}\right) + (1 - \gamma_n) \phi\left(\frac{y - s_x \mu_g}{\sigma_g}\right) \right] + ((1 - \omega)(1 - \pi) + \omega)g(y; \mu_g, \sigma_g),$$

where π_n is given in (5) and g is given in (17).

Specification of Model CS₄: Different Guessing on False- and True-Probe Trials

The model is the same as Model CS₃ with different specifications of guessing for false-probe and true-probe trials. For false probe trials, g is

$$g(y; \mu_f, \sigma_f) = \frac{1}{2} \phi\left(\frac{y - \mu_f}{\sigma_f}\right) + \frac{1}{2} \phi\left(\frac{y + \mu_f}{\sigma_f}\right).$$

For the true-probe trials

$$g(y; \mu_t, \sigma_t) = \frac{1}{2} \phi\left(\frac{y - \mu_t}{\sigma_t}\right) + \frac{1}{2} \phi\left(\frac{y + \mu_t}{\sigma_t}\right)$$

The model reduces to CS₃ if $\mu_v = \mu_f$ and $\sigma_v = \sigma_f$.

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Footnotes

¹Smoothing was done by Gaussian kernel smoothing with default settings in R.

²The density of an inverse gamma distribution is given by

$$f(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)x^{\alpha+1}} \exp(-\beta/2x),$$

where α and β serve as shape and scale parameters, respectively.

Table 1

Model selection results for Experiments 1 through 3.

	Distributed Resources							Discrete Slots			Coarse Encoding		
	\mathcal{R}_1	\mathcal{R}_2	\mathcal{R}_3	\mathcal{R}_4	\mathcal{R}_5	\mathcal{R}_6	\mathcal{R}_7	\mathcal{S}_1	\mathcal{S}_2	\mathcal{S}_3	\mathcal{CR}	\mathcal{CS}_1	\mathcal{CS}_2
Experiment 1 (24 Participants)													
AIC	0	0	0	0	0	0	0	0	1	1	0	14	8
BIC	0	0	0	0	0	0	0	1	1	5	0	14	3
Sum AIC	3829	3876	2726	2716	3313	3370	3504	896	564	709	1933	44	0
AIC w/ Lapse	0	0	0	0	0	0	0	1	8	1	0	8	6
Experiment 2 (26 Participants)													
AIC	0	0	0	1	0	0	0	0	5	2	4	2	12
BIC	0	0	0	2	0	1	0	0	5	8	3	3	4
Sum AIC	2290	2453	1034	1114	1565	1650	1666	1519	211	480	324	679	0
AIC w/ Lapse	0	0	0	1	0	0	0	0	3	1	2	2	17

Notes. **AIC** & **BIC**: The number. of participants for which the model was selected by AIC and BIC, respectively. **Sum AIC**: Increase in AIC values over winning model summed across participants. **AIC w/ Lapse**: Number. of participants for which the model was selected by AIC when 5% of responses were assumed random guesses from a lapse in attention. $\mathcal{R}_1, \dots, \mathcal{CS}_2$ are resource, slot, and coarse encoding models as defined in the text and Appendix.

Table 2

Median parameter estimates

	ω	K	σ_1	σ_3	σ_6	γ_3	γ_6	μ_g	σ_g
Experiment 1	.02	2.7	8.1	11.2	14.3	.56	.41	30.9	9.4
Experiment 2	≈ 0	3.6	5.8	10.8	14.6	.86	.47	23.2	16.5
Experiment 3	.03	2.4	12.3	13.2	17.8	.39	.25	34.2	12.6

Notes. ω = Rate of Attention Lapse, K = capacity. $\sigma_1, \dots, \sigma_6$ = Standard deviation of responses from memory for set sizes of 1, 3, and 6, respectively. γ_3, γ_6 = fine-encoding rate for set sizes 3 and 6, respectively. μ_g, σ_g = mean and standard deviation for a guessing band, respectively.

Table 3

Model-fitting results for Experiments 4a and 4b.

	Distributed Resources			Discrete Slots			Coarse Encoding		
	\mathcal{R}_{1a}	\mathcal{R}_{3a}	\mathcal{R}_{5a}	\mathcal{S}_{1a}	\mathcal{S}_{2a}	\mathcal{S}_{3a}	\mathcal{CR}_a	\mathcal{CS}_{1a}	\mathcal{CS}_{2a}
Experiment 6a (19 Participants)									
AIC	0	0	0	0	0	0	4	8	7
BIC	0	0	0	1	1	0	4	8	5
Sum AIC	2233	1462	2279	446	101	440	1054	222	0
AIC w/ 5% Lapse	0	0	0	0	2	0	1	15	1
Experiment 6b (25 Participants)									
AIC	0	0	0	1	3	0	3	12	6
BIC	0	0	0	3	3	2	3	9	5
Sum AIC	4045	3277	4479	491	117	472	1598	240	0
AIC w/ 5% Lapse	0	0	0	1	3	0	2	13	6

Notes. **AIC** & **BIC**: The number. of participants for which the model was selected by AIC and BIC, respectively. **Sum AIC**: Increase in AIC values over winning model summed across participants. **AIC w/ Lapse**: Number. of participants for which the model was selected by AIC when 5% of responses were assumed random guesses from a lapse in attention. $\mathcal{R}_2, \dots, \mathcal{CS}_{2a}$ are resource, slot, and coarse encoding models as defined in the text and Appendix.

Figure Captions

Figure 1. Experimental Paradigm. **A.** Items are rings, and the to-be-remembered attribute is the angle of the gem (filled small circle). Shown is a stimulus with a study angle of -45° . **B.** Trial structure: A trial consists of the presentation of several to-be-remembered items followed by a short delay. Participants are asked to produce the study angle of one item (the green one in the figure) by moving the gem. The ideal response is shown as an open circle.

Figure 2. Predicted data patterns. **A.** Basic distributed-resources model. **B.** Basic discrete-slots model with guessing centered at 0° . **C.** Basic discrete-slots model with a bimodal guessing pattern. **D.** A mixture of finely and coarsely encoded items.

Figure 3. Results from Experiment 1. **A.** Response angle plotted as a function of study angle and set size. The vertical dotted line for the 6-item condition shows study angles at least 50° . The arrow for the 1-item condition highlights a response that has inordinate leverage on parameter estimates in some of the discussed models. **B.** Response angles for individual participants to study angles of at least 50° . The bimodal pattern is seen for the vast majority of participants, and this pattern indicates that participants guessed with stereotypical leftward and rightward angles.

Figure 4. Coarse-Encoding Discrete Slot account of Experiments 1, 2, and 3. Rows are for different ranges of study angles; columns are for different experiments. Blue histograms denote the data from the six-item condition, the red distribution are responses from memory; and the green distribution are responses from coarse encoding and guessing. These later responses are not a function of stimulus angle. The dashed lines are the model's prediction.

Figure 5. Individual and median estimates for select parameters in Experiments 1, 2, and

3. **A.** Estimates for the probability of guessing in Model \mathcal{R}_8 . **B.** Estimates of guessing ($\omega + (1 - \omega)(1 - \pi_n)$) from Model \mathcal{CS}_3 . **C.** Estimates for Probability of Coarse Encoding in Model \mathcal{CS}_3 . Individual participant estimates are plotted in grey, with the medians plotted in red.

Figure 6. Results from Experiment 2: Response angle plotted as a function of study angle and set size. The results of Experiment 2 are similar to those of Experiment 1.

Figure 7. Results from Experiment 3: Response angle plotted as a function of study angle and set size. Responses on false-probe trials are plotted, in red, on the right side of the plot. These responses appear to form bands similar to the guessing bands observed in responses to true-probe trials.

Figure 8. Results of Experiment 4a. Response angle plotted as a function of study angle and set size. Responses on false-probe trials are plotted in red on the right side of the plot. **A., B.** Results for leftward-prevalent and rightward-prevalent conditions, respectively.

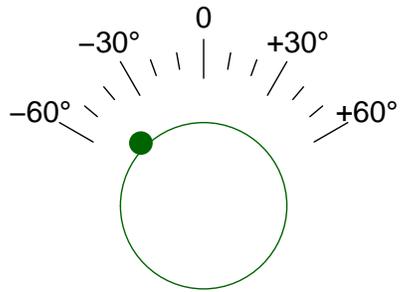
Figure 9. Density plots of participant responses for a small range of study angles. Density estimates were obtained by passing a Gaussian kernel across observations. **A & C.** Study angles are between -40° and -30° for Experiments 6a and 6b, respectively. **B & D.** Study angles are between 30° and 40° for Experiments 6a and 6b, respectively. The densities show a dramatic ability to selectively influence the modes of a mixture with a base-rate manipulation. Such selective influence is strong evidence for guessing.

Figure 10. Box plots of guessing and capacity parameter estimates from Model \mathcal{CS}_{4a} for leftward- and rightward-prevalence conditions, respectively. The plotted parameters for the first two panel is η , the probability weight of the leftward mode. As can be seen, this weight is high for the leftward-prevalent condition and low for the rightward prevalent

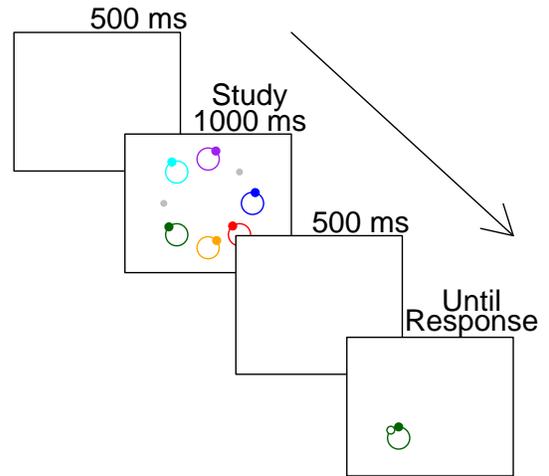
condition, and this pattern holds for both true-probe and false-probe trials. The last panel shows that the capacity estimate k is seemingly unaffected by the prevalence manipulation.

Working-Memory Performance, Figure 1

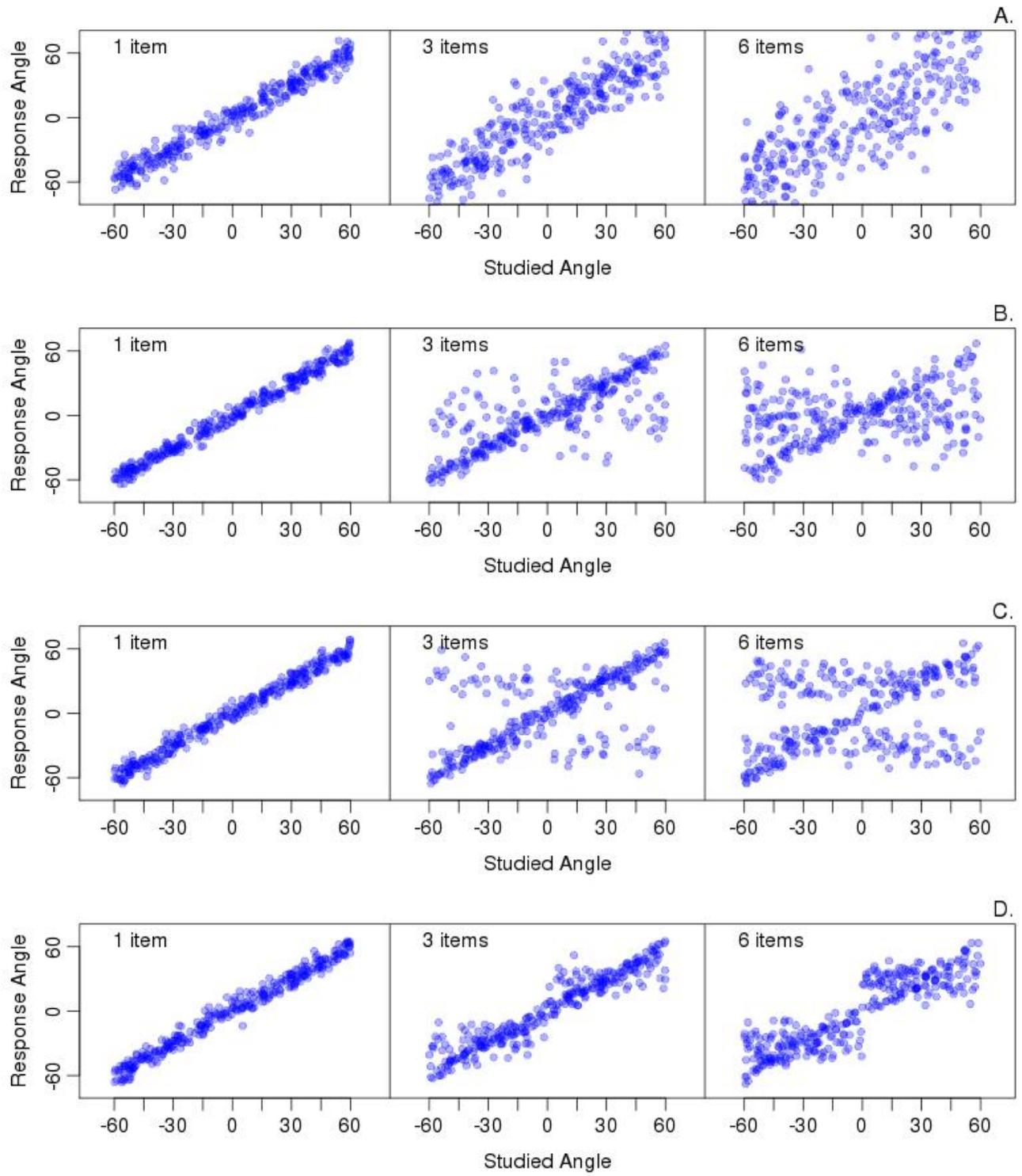
A.



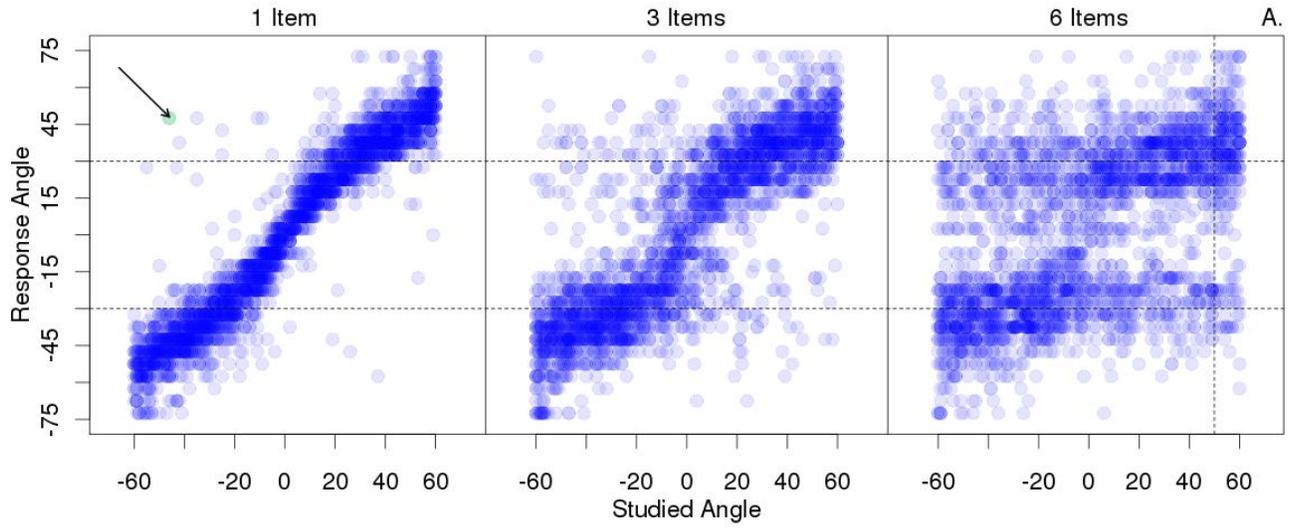
B.



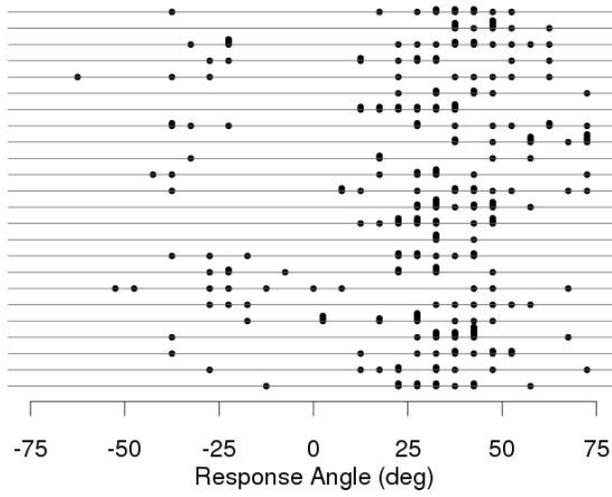
Working-Memory Performance, Figure 2



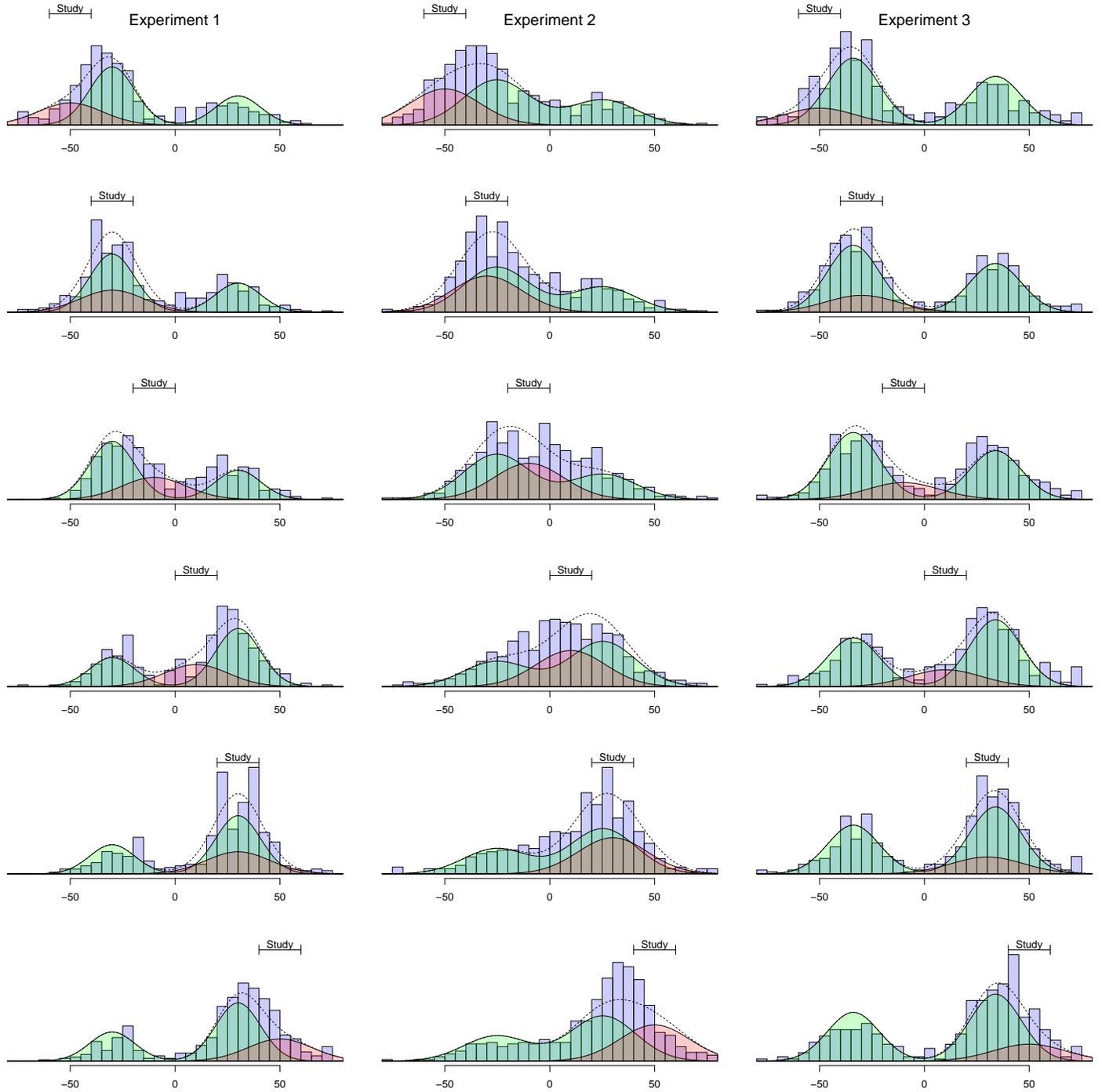
Working-Memory Performance, Figure 3



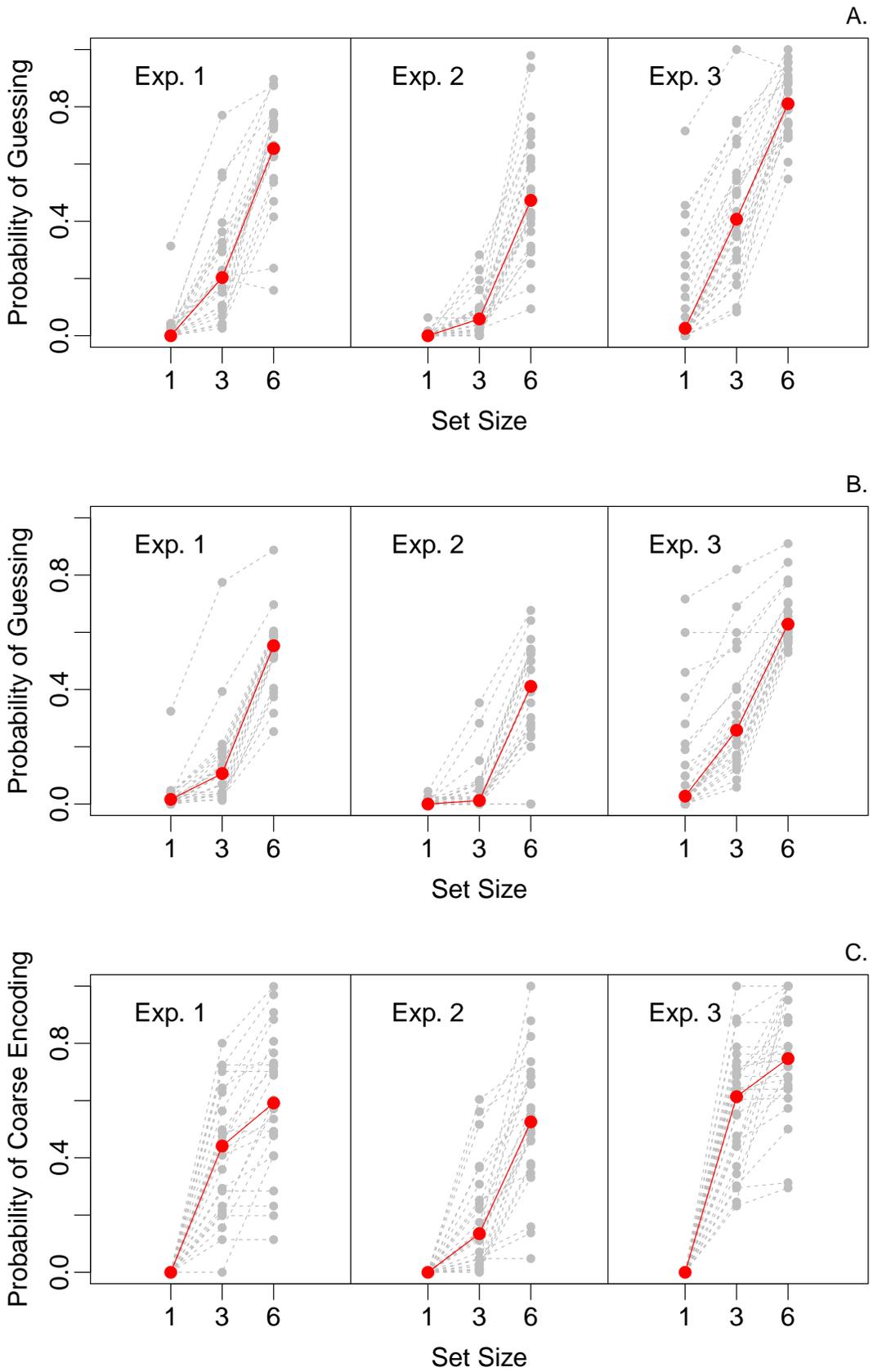
B.



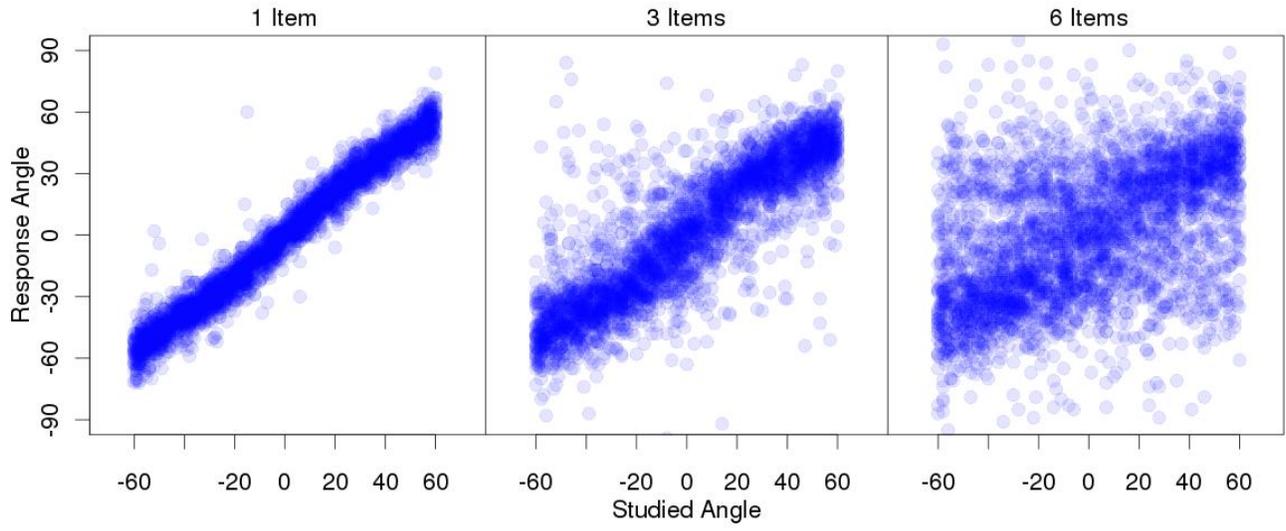
Working-Memory Performance, Figure 4



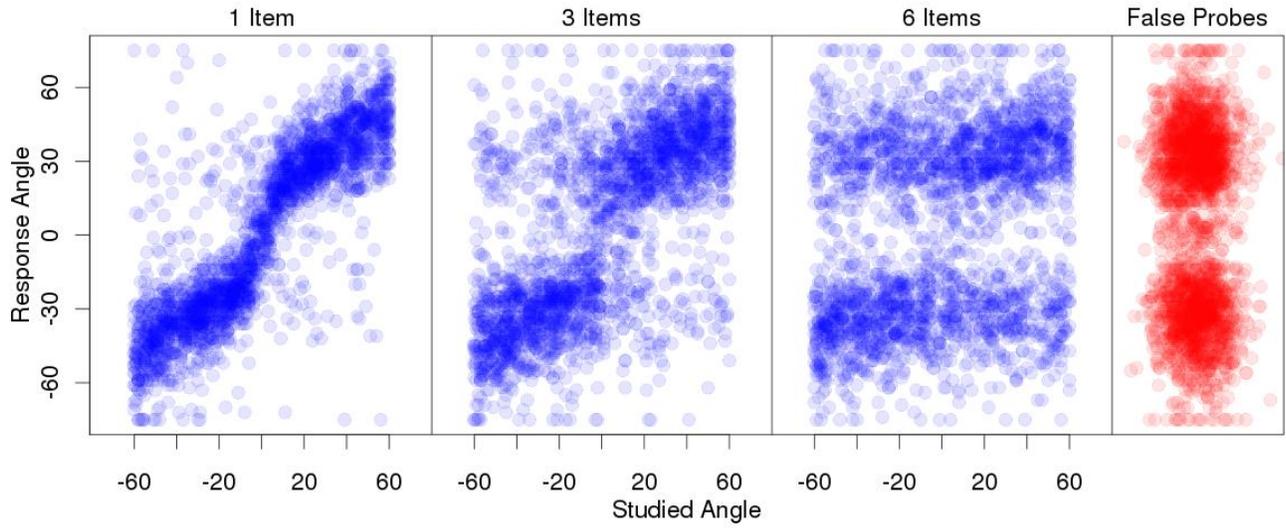
Working-Memory Performance, Figure 5



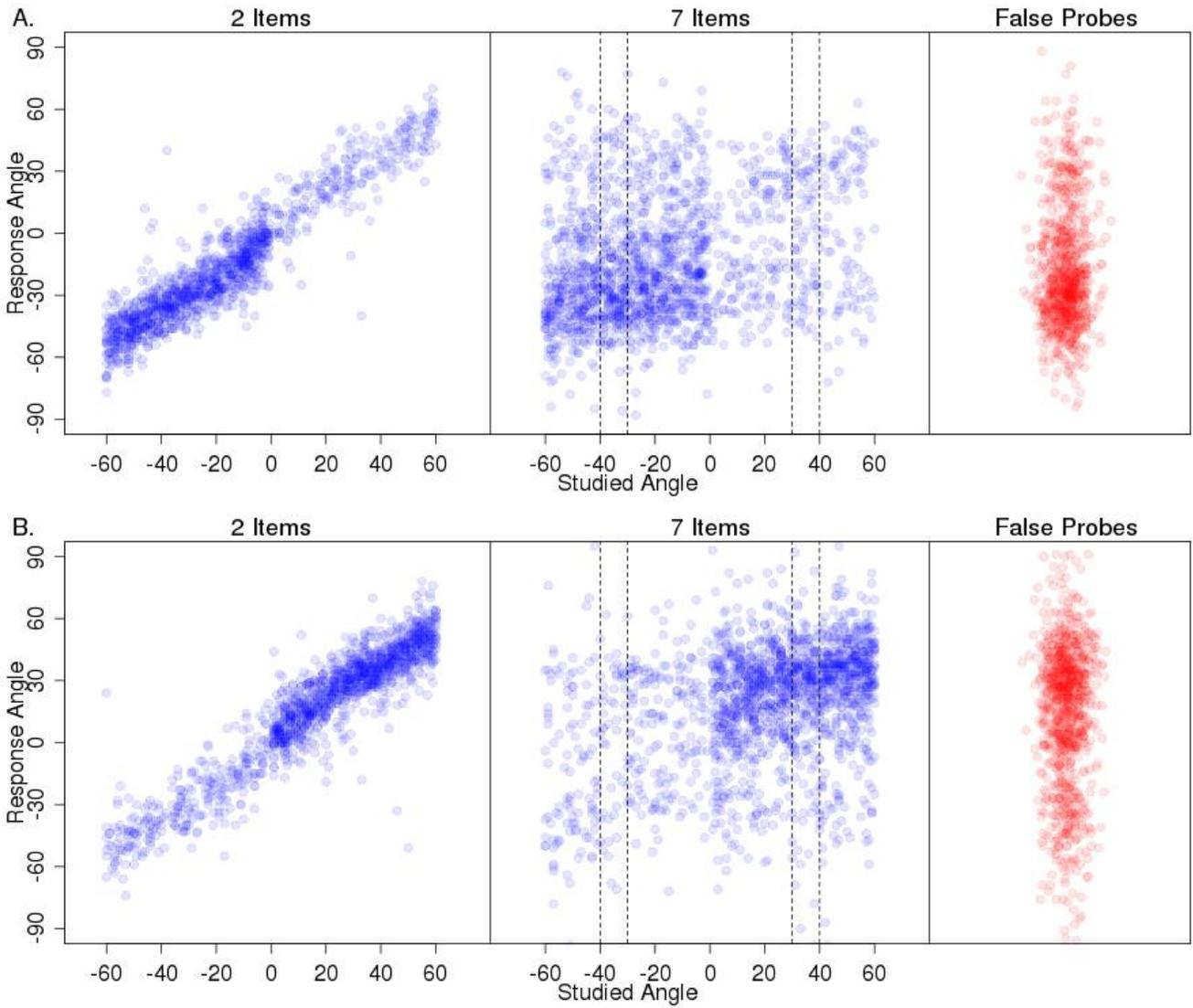
Working-Memory Performance, Figure 6

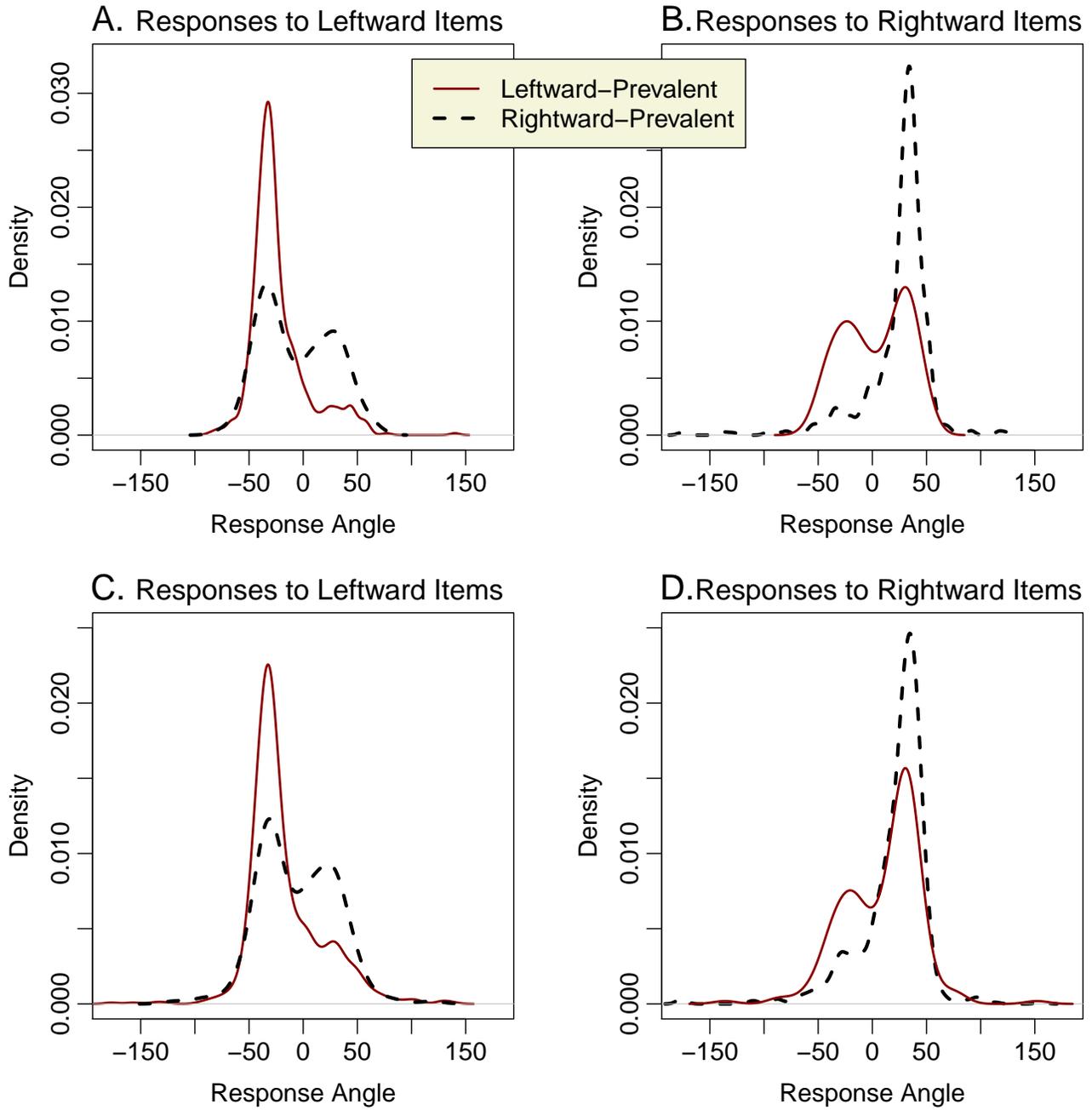


Working-Memory Performance, Figure 7



Working-Memory Performance, Figure 8





Working-Memory Performance, Figure 10

